Theoretical considerations for a jet simulation with spin

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THEORETICAL CONSIDERATIONS FOR A JET SIMULATION WITH SPIN[1]

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Abstract

A first part lists basic rules, taken from the string- and multiperipheral models, that a recursive quark fragmentation model should obey. A second part describes spin effects given by the classical "string + 3P0" mechanism of quark-antiquark pair creation, in pseudoscalar and vector meson production: Collins effect, jet handedness and "hidden spin" effects in unpolarized experiments. The last part constructs a recursive quantum-mechanical model of spin-dependent fragmentation. In a "ab initio" approach an integral equation must be solved as a preliminary task. With a "renormalized input", this task is reduced to an ordinary integration. A spin-dependent generalization of the symmetric Lund model is obtained.

1 Introduction

A jet model which takes into account the quark spin degree of freedom must start with *quantum amplitudes* rather than probabilities. A "toy model" [1] using Pauli spinors and inspired from the multiperipheral model and the classical *string* $+^3P_0$ mechanism [2, 3] followed this principle. Collins- and *longitudinal jet handedness* [4] effects were generated. However hadron mass-shell constraints were ignored. These constraints are satisfied in an improved model [5], which is a *symmetric-Lund* model endowed with spin factors. In the *ab initio* approach of [5] the inputs are quark *propagators* and quark-hadron *vertices* derived from a string action. The recursive *splitting function* is obtained by solving an integral equation. We will show that, starting from a *renormalized input*, this preliminary task is replaced by an ordinary integration.

Section 2 lists the rules and approximations of a *bona fide* recursive jet model. Spin effects produced by the classical *string* $+^3P_0$ mechanism or the "toy model" are sketched in Sec.3. The next sections develop the model of Ref. [5] in three stages: the *ab initio* approach, the *renormalized input* approach and the application with string anplitudes.

2 Rules and approximations for a recursive model

We take the example of $W^\pm$ decay into $\mathrm{q_A} + \bar{\mathrm{q}}_\mathrm{B}$ and no gluon (lower part of Fig.1-left) followed by a hadronisation into mesons and no baryon (upper part of Fig.1-left),

$$\mathrm{q_A} + \bar{\mathrm{q}}_\mathrm{B} \to \mathrm{h}_1 + \mathrm{h}_2 ... + \mathrm{h}_N \,. \tag{1}$$

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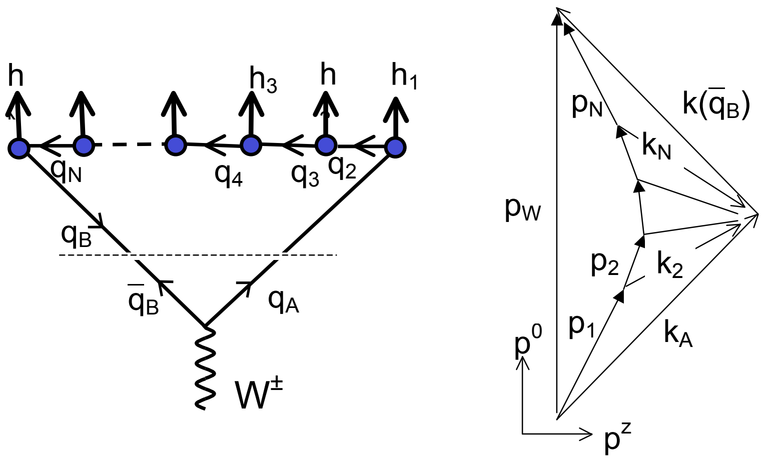


Figure 1: Left: quark-diagram of a hadronic decay of $W^{\pm}$. Right: associated momentum diagram, projected on the (p^0, p^z) plane.

In the multiperipheral picture, (1) is decomposed in recursive *quark splittings*

$$\mathrm{q}_1 \to \mathrm{h}_1 + \mathrm{q}_2\,, \quad \mathrm{q}_2 \to \mathrm{h}_2 + \mathrm{q}_3\,, \cdots\cdots \quad \mathrm{q}_N \to \mathrm{h}_N + \mathrm{q}_\mathrm{B}\,, \tag{2}$$

with $\mathrm{q}_1 \equiv \mathrm{q}_\mathrm{A}$; h_n is the meson of *rank* $n \le N$; $\mathrm{q}_\mathrm{B} \equiv \mathrm{q}_{N+1}$ is the charge conjugate of $\bar{\mathrm{q}}_\mathrm{B}$ and "propagates backward in time".

Factorization. We assume the approximate *probability* convolution

$$\mathcal{P}_\mathrm{event} \simeq \int d\Omega\, \frac{d\mathcal{P}(W^{\pm} \to \mathrm{q}_\mathrm{A}\bar{\mathrm{q}}_\mathrm{B})}{d\Omega} \times \mathcal{P}(\mathrm{q}_\mathrm{A} + \bar{\mathrm{q}}_\mathrm{B} \to \mathrm{h}_1 + \mathrm{h}_2 ... + \mathrm{h}_N)\,. \tag{3}$$

$\mathcal{P}_\mathrm{event}$ is the exclusive N-particle distribution of the whole event. $d\mathcal{P}/d\Omega$ is the angular distribution of the quark momentum $\mathbf{k}_\mathrm{A}$ in the $W^{\pm}$ rest frame. The last factor is the exclusive N-particle distribution of reaction (1). $\mathbf{k}_\mathrm{A}/|\mathbf{k}_\mathrm{A}| = \hat{\mathbf{z}}$ defines the *jet axis*. In a more rigorous approach the convolution should bear on the *amplitudes*. k_A is an internal momentum of the loop diagram of Fig.1-left and $\mathcal{P}_\mathrm{event}$ is a double integral: in k_A for the amplitude and in k'_A for the complex conjugate amplitude. Factorization (3) ignores the pure quantum-mechanical quantity $k_\mathrm{A} - k'_\mathrm{A}$.

Multiperipheral dynamics. Each splitting conserves 4-momentum: $k_n = p_n + k_{n+1}$. These relations are exhibited in the momentum diagram of Fig.1-right. A basic ingredient of the multiperipheral model is the cutoff in the quark virtualities $-k^2$. It implies:

- a cutoff in $|k^+k^-| \equiv (k^0 + k^z)\,|k^0 - k^z|$, which insures the approximate ordering of $\mathrm{h}_1, \mathrm{h}_2, \cdots \mathrm{h}_N$ in rapidity and the *leading particle effect* (or *favored fragmentation*).

- a cutoff in $\mathbf{k}_\mathrm{T}$ leading to the *Local Compensation of Transverse Momenta* (LCTM) [6]. It leads to a cutoff in $\mathbf{p}_\mathrm{T}$ of the hadrons[2,3].

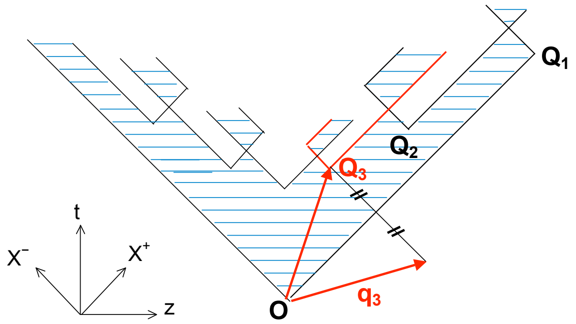


Figure 2: Relation (4) between the quark momentum q_3 in the multiperipheral picture and the point Q_3 where the $q_3\bar{q}_3$ pair is created in the classical string fragmentation model with $m_q = 0$, $\mathbf{k}_T = 0$.

Ladder approximation. A same hadronic final state can be obtained with several multiperipheral diagrams which differ by permutations. In the *ladder approximation* the interferences between these diagrams are neglected. Most often only one diagram is important, the others having rank ordering too far from the rapidity ordering.

String dynamics. The same properties are found in the String Fragmentation Model. Fig.2 represents the world sheet of the massive string or *dart* stretched by q_A and $\bar{q}_B$ and decaying into hadrons, in a classical 1+1 dimensional model with massless quarks. It is a particular type of quark multiperipheral model, if one orders the Q-corners according to the null-plane time variable $X^- = t - z$ and make the correspondance[4]

$$t(Q_n) - t(O) = k_n^z/\kappa\,, \quad z(Q_n) - z(O) = k_n^0/\kappa\,, \tag{4}$$

where $\kappa \simeq 1$ GeV/fm is the string tension (hereafter we take $\kappa = 1$). For a string breaking point Q the condition that there is no other breaking in its past cone leads to the suppression of large $(OQ)^2 \equiv -k^+k^-$ by a factor

$$\exp\big(- b\,|k^+k^-|\big) \tag{5}$$

where $2b$ is the string "fragility" in units $\kappa = 1$. Quarks with masses and transverse momenta are thought to be produced by a tunneling mechanism similar to the Schwinger one for e^+e^- creation in strong electric field. It provides the k_T cutoff factor

$$\exp[-\pi(m_q^2 + k_T^2)/\kappa]. \tag{6}$$

3 Properties of the classical *string* + $^3P_0$ mechanism

Fig.3 depicts the decay of the dart as if all Q_n where at equal time. Assuming that a $q_n\bar{q}_n$ pair is created at Q_n in the $^3P_0$ state and with zero 4-momentum, one predicts a correlation between the antiquark polarization $\bar{\mathbf{S}}_n$ and transverse momentum $\bar{\mathbf{k}}_{n,T}$: $\langle\bar{\mathbf{k}}_n \cdot (\hat{\mathbf{z}} \times \bar{\mathbf{S}}_n)\rangle$ is positive. A similar effect is predicted in atomic physics [7].

[2]The converse is not true: the $\mathbf{p}_T$ cutoff alone, used in some models, does not lead to a $\mathbf{k}_T$ cutoff.
[3]The symmetric Lund splitting function reinforces the $\mathbf{p}_T$ cutoff by the factor $\exp[-b(m_h^2 + p_T^2)/Z]$.
[4]k = *canonical* quark momentum = mechanical momentum + string momentum flow through OQ.

Case where $\mathrm{h}_1, \mathrm{h}_2, \ldots$ **are pseudoscalar mesons.** In that case q_n and $\bar{\mathrm{q}}_{n+1}$ forming h_n have antiparallel spins. Combined with the $\langle\bar{\mathbf{k}}\cdot(\hat{\mathbf{z}}\times\bar{\mathbf{S}})\rangle$ correlations it gives:

− a Collins effect toward $\mathbf{S}_1 \times \hat{\mathbf{z}}$ for the "favored" meson h_1,
− Collins effects of alternate sides for the next mesons,
− a large Collins effect for h_2,
− *Relative* Collins Effects (or *IFF*) larger than from "single-Collins" + LCTM alone.

Case where h_1 **is a leading vector meson.** In a vector meson of linear polarization $\mathbf{A}$ (being known from the decay products), the q and $\bar{\mathrm{q}}$ polarizations are symmetrical about the plane perpendicular to $\mathbf{A}$ (Fig.3b). Let us consider a 1$^{\mathrm{rst}}$-rank vector meson:
− if $\mathbf{A} \parallel \hat{\mathbf{z}}$ the Collins asymmetry is opposite to that of a leading pseudoscalar meson,
− if $\mathbf{A} \perp \hat{\mathbf{z}}$ the Collins asymmetry is in the azimuth $2\phi(\mathbf{A}) - \phi(\mathbf{S}_1) - \pi/2$,
− if both $A_z \neq 0$ and $\mathbf{A}_\mathrm{T} \neq 0$ and if q_1 is helicity-polarized, $S_{1z}\, A_z\, \mathbf{A}\cdot\langle\hat{\mathbf{z}} \times \mathbf{p}\rangle$ is positive. This is a *longitudinal jet-handeness* [4] effect.

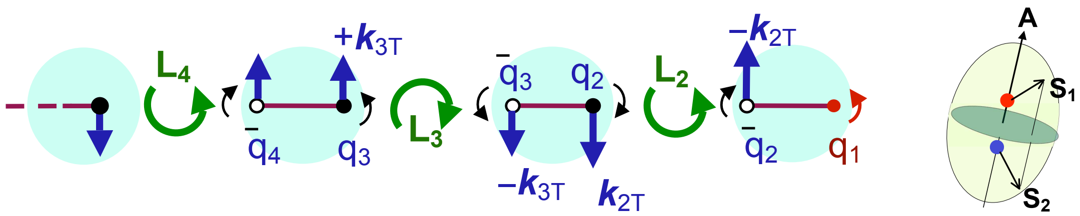
These three effects are reproduced by the "toy model". They correspond respectively to lines 3, 5 and 6 of Eq.(27) of [1]. On the average, the Collins effect is -1/3 that of the pseudoscalar meson [8].

Hidden spin effects. Whether q_A is polarized or not, the $\langle\bar{\mathbf{k}}\cdot(\hat{\mathbf{z}}\times\bar{\mathbf{S}})\rangle$ correlation of the *string* $+^3P_0$ mechanism has an impact on the $\mathbf{p}_\mathrm{T}$ distribution of the rank≥ 2 mesons:
− for a pseudoscalar meson, $\langle\mathbf{p}_\mathrm{T}^2\rangle_\mathrm{meson} > 2\,\langle\mathbf{k}_\mathrm{T}^2\rangle_\mathrm{quark}$,
− for a vector meson linearly polarized along $\hat{\mathbf{z}}$, $\langle\mathbf{p}_\mathrm{T}^2\rangle_\mathrm{meson} < 2\,\langle\mathbf{k}_\mathrm{T}^2\rangle_\mathrm{quark}$,
− for a vector meson linearly polarized along $\hat{\mathbf{x}}$, $\langle p_x^2\rangle < 2\,\langle\mathbf{k}_\mathrm{T}^2\rangle < \langle p_y^2\rangle$.
On the average, $\langle\mathbf{p}_\mathrm{T}^2\rangle_\text{V-meson} < \langle\mathbf{p}_\mathrm{T}^2\rangle_\text{PS-meson}$. These "hidden spin" effects allow an unexpensive test of the *string*$+\,^3P_0$ mechanism (note that the *Schwinger mechanism* predicts no $\langle\bar{\mathbf{k}}\cdot(\hat{\mathbf{z}}\times\bar{\mathbf{S}})\rangle$ correlation [9]). At least they suggest that quark spin plays a role even in unpolarized experiments and should be included in any jet model.

4 The *ab initio* approach

The starting point is the multiperipheral hadronization amplitude

$$\langle k_\mathrm{B}, s_\mathrm{B}|\mathcal{M}_N\{\mathrm{q_A\bar{q}_B} \to \mathrm{h_1 h_2 \cdots h}_N\}|k_\mathrm{A}, s_\mathrm{A}\rangle = \\ \langle k_\mathrm{B}, s_\mathrm{B}|\,\mathcal{D}\{\mathrm{q_B}\}\,\mathcal{V}\{\mathrm{q_B}, \mathrm{h}_N, \mathrm{q}_N\}\cdots\mathcal{D}\{\mathrm{q_3}\}\,\mathcal{V}\{\mathrm{q_3}, \mathrm{h_2}, \mathrm{q_2}\}\,\mathcal{D}\{\mathrm{q_2}\}\,\mathcal{V}\{\mathrm{q_2}, \mathrm{h_1}, \mathrm{q_A}\}\,\mathcal{D}\{\mathrm{q_A}\}\,|k_\mathrm{A}, s_\mathrm{A}\rangle\,. \quad (7)$$



(a) (b)

Figure 3: (a) string decay into pseudoscalar mesons with the *string*$+^3P_0$ mechanism. (b) spin correlation of the quark and antiquark in a vector meson linearly polarized along $\mathbf{A}$.

$|k_{\rm B}, s_{\rm B}\rangle$ is the negative energy state whose hole is $|k(\bar{\rm q}_{\rm B}), s(\bar{\rm q}_{\rm B})\rangle$. Inside curly brakets, $\{{\rm q}\} = (f, k)$ gathers the quark flavor f and 4-momentum k. For a meson $\{{\rm h}\} = (h, p, s_{\rm h})$ gathers the species h, the 4-momentum and the spin state. The quark propagator $\mathcal{D}\{{\rm q}\} \equiv \mathcal{D}(f, k)$ and the vertex function $\mathcal{V}\{f', h, f\} \equiv \mathcal{V}_{f',h,f}(k', k)$ are the *inputs* of the model. In a *step-by-step* covariant model, $|k_{\rm A}, s_{\rm A}\rangle$ and $|k_{\rm B}, s_{\rm B}\rangle$ would be Dirac spinors and $\mathcal{D}$ and $\mathcal{V}$ would be 4×4 matrices, *e.g.,* $\mathcal{D}\{{\rm q}\} = D(f, k^2)\,(m_f + \gamma\cdot k)$. However Lorentz covariance is required only *globally* for the whole process of Fig.1. Together with P and C conservation, this requires the invariance of $\mathcal{M}$ under

– (a) rotations about $\hat{\mathbf{z}}$,
– (b) Lorentz transformations along $\hat{\mathbf{z}}$,
– (c) reflection about any plane containing $\hat{\mathbf{z}}$,
– (d) *quark chain reversal* or "left-right symmetry" [2], *i.e.,* interchanging ${\rm q}_{\rm A}$ and $\bar{\rm q}_{\rm B}$.

These invariances can be realized with *Pauli* spinors. For instance, we will take [1]

$$\mathcal{D}\{{\rm q}\} = D(f, k^+k^-, \mathbf{k}_{\rm T}^2)\,(\mu_f + \sigma_z\,\sigma\cdot\mathbf{k}_{\rm T})\,. \tag{8}$$

Doing so, we do not take into account the whole information (2 q-bits) carried by an off-mass-shell Dirac spinor. We leave this question for further studies.

Hadronization "cross section" of quark ${\rm q}_n$. In the ladder approximation one can define the hadronization "cross section" of an initial or intermediate polarized quark ${\rm q}_n$,

$$\mathcal{H}\{\bar{\rm q}_{\rm B} + \uparrow{\rm q}_n \to X\} = {\rm Tr}\,\mathcal{R}\{{\rm q}_n\}\,\rho\{{\rm q}_n\}\,, \tag{9}$$

where $\rho\{{\rm q}_n\} = (\mathbf{I} + \sigma\cdot\mathbf{S}_n)/2$ is the spin density matrix of ${\rm q}_n$,

$$\mathcal{R}\{{\rm q}_n\} = \frac{1}{2}\sum_{N\geq n}\int d\{{\rm h}_n\}\cdots d\{{\rm h}_N\}\,\mathcal{M}^\dagger_{N-n}\,\mathcal{M}_{N-n}\,\delta^4[p_n + \cdots + p_N - k_{\rm A} - k(\bar{\rm q}_{\rm B})] \tag{10}$$

and $\int d\{{\rm h}\}\cdots$ stands for $\sum_h\sum_{s_{\rm h}}\int {\rm d}^3\mathbf{p}/p^0\cdots$. We are interested in the ${\rm q}_{\rm A}$ fragmentation region, that is why we will took $\bar{\rm q}_{\rm B}$ unpolarized. $\mathcal{R}\{{\rm q}\}$ obeys the *ladder* integral equation (illustrated by Fig.4):

$$\mathcal{R}\{{\rm q}\} = \int d\{{\rm h}\}\,T^\dagger\{{\rm q}', {\rm h}, {\rm q}\}\,\mathcal{R}\{{\rm q}'\}\,T\{{\rm q}', {\rm h}, {\rm q}\} + \sum_{h,\,s_{\rm h}}\mathcal{M}_1^\dagger\,\mathcal{M}_1\,\delta[(k - k_{\rm B})^2 - m_{\rm h}^2] \tag{11}$$

with $T\{{\rm q}', {\rm h}, {\rm q}\} \equiv \mathcal{V}\{{\rm q}', {\rm h}, {\rm q}\}\,\mathcal{D}\{{\rm q}\}$. At large $m_{\rm X}^2 \simeq |k_{\rm B}^-|\,k^+$,

$$\mathcal{R}\{{\rm q}\} \simeq \mathcal{B}\{{\rm q}\}\,(m_{\rm X}^2)^{\alpha_{\rm R}}\,, \tag{12}$$

$$\mathcal{B}\{{\rm q}\} = \beta(f, \mathbf{k}_{\rm T}^2)\,[1 + A(f, \mathbf{k}_{\rm T}^2)\,\sigma\cdot\tilde{\mathbf{n}}(\mathbf{k})]\,, \tag{13}$$

with $\tilde{\mathbf{n}}(\mathbf{k}) \equiv \hat{\mathbf{z}}\times\mathbf{k}/|\hat{\mathbf{z}}\times\mathbf{k}|$. In ordinary multiperipheral models $\alpha_{\rm R}$ and $\mathcal{B}\{{\rm q}\}$ are the intercept and residue of the *output Regge trajectory.* $A(f, \mathbf{k}_{\rm T}^2)$ is the single-spin asymmetry of $\uparrow{\rm q} + \bar{\rm q}_{\rm B} \to {\rm X}$. $A(f, 0) = 0$. $\mathcal{B}\{{\rm q}\}$ is semi-positive definite: $\beta > 0$, $|A| \leq 1$.

Recursive Monte-Carlo algorithm. Suppose that we have already generated $n-1$ steps of (2) and recorded the density matrix $\rho\{{\rm q}_n\}$. The simulation of the next step $\uparrow{\rm q}_n \to {\rm h}_n + \uparrow{\rm q}_{n+1}$ (hereafter rewritten $\uparrow{\rm q} \to {\rm h} + \uparrow{\rm q}'$) proceeds in two sub-steps:

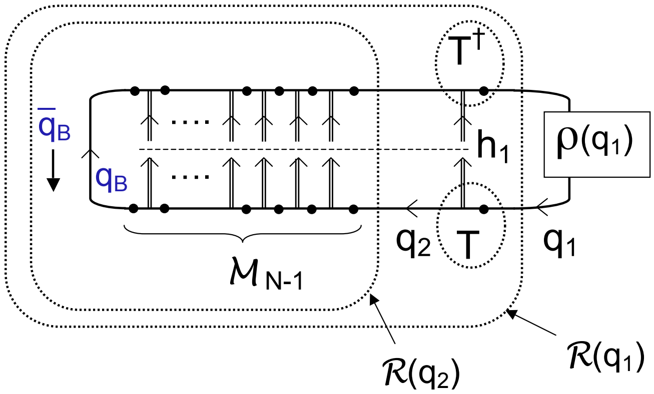


Figure 4: Ladder unitarity diagram associated to Eqs.(9-11) with n=1, $\mathrm{q} = \mathrm{q}_1$, $\mathrm{q}' = \mathrm{q}_2$. Black bullets represent quark propagators. The summation over N is understood.

1) generate the species and momentum of h**.** From Eqs.(11-12) the type and momentum distribution of the next-rank particle is proportional to

$$d\mathcal{H}\{\bar{\mathrm{q}}_\mathrm{B}+\uparrow\mathrm{q}\} = \frac{dZ\, d^2\mathbf{p}_\mathrm{T}}{Z}\, |k_\mathrm{B}^-\, k'^+|^{\alpha_\mathrm{R}} \sum_{s_\mathrm{h}} \mathrm{Tr}\left[\mathcal{B}\{\mathrm{q}'\}\; T\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\; \rho\{\mathrm{q}\}\; T^\dagger\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\right], \tag{14}$$

with $Z \equiv p^+/k^+,\ k' = k - p$.

2) calculate the polarization of $\uparrow\mathrm{q}'$. It is given by

$$\rho\{\mathrm{q}'\} = \left[\sum_{s_\mathrm{h}} T\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\; \rho\{\mathrm{q}\}\; T^\dagger\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\right] / \,\mathrm{Tr}\,[\mathrm{idem}]\,. \tag{15}$$

If h has nonzero spin and one wants to simulate its decay, a more complicated algorithm is needed, following the rules of [11] (see also Sec.5.1 of [12]).

In this *ab initio* approach one must calculate α_R and the functions $\beta(f, \mathbf{k}_\mathrm{T}^2)$ and $A(f, \mathbf{k}_\mathrm{T}^2)$ from the integral equation (11), as a preliminary numerical task.

5 The *renormalized input* approach

The physical properties (*e.g.*, the multi-particle distributions) are unchanged by two kinds of "renormalization" of the propagators and vertices:

$$\begin{aligned} &(a) \quad \text{new } \mathcal{D}\{\mathrm{q}\} = |k^- k^+|^\lambda\, \mathcal{D}\{\mathrm{q}\}\,, \qquad \text{new } \mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\} = |k'^+ k^-|^\lambda\, \mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\} \\ &(b) \quad \text{new } \mathcal{D}\{\mathrm{q}\} = \Lambda\{\mathrm{q}\}\, \mathcal{D}\{\mathrm{q}\}\, \Lambda\{\mathrm{q}\}\,, \quad \text{new } \mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\} = \Lambda^{-1}\{\mathrm{q}'\}\, \mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\, \Lambda^{-1}\{\mathrm{q}\}\,, \end{aligned} \tag{16}$$

where $\Lambda\{\mathrm{q}\} \equiv \Lambda(f, \mathbf{k}_\mathrm{T})$ is a matrix in spin space. Under (a) α_R is shifted by 2λ. Under (b), *new* $\mathcal{B}\{\mathrm{q}\} = \Lambda^\dagger\{\mathrm{q}\}\, \mathcal{B}\{\mathrm{q}\}\, \Lambda\{\mathrm{q}\}$. Let us combine (a) and (b) with $\lambda = -\alpha_\mathrm{R}/2$ and $\Lambda = \mathcal{B}^{-\frac{1}{2}}\, (\mathcal{D}^\dagger/\mathcal{D})^{\frac{1}{4}}$ (these matrices commute). Then *new* $\alpha_\mathrm{R} = 0$, *new* $\mathcal{R}\{\mathrm{q}\} = \mathbf{I}$. Taking the renormalized $\mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\}$ as unique input, the renormalized propagator is obtained from (11):

$$\mathcal{D}\{\mathrm{q}\} = U^{-\frac{1}{2}}\{\mathrm{q}\} \quad \text{with} \quad U\{\mathrm{q}\} \equiv \int d\{\mathrm{h}\}\; \mathcal{V}^\dagger\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\, \mathcal{V}\{\mathrm{q}',\mathrm{h},\mathrm{q}\}\,. \tag{17}$$

The preliminary task is now to evaluate (17). It is much easier than solving the integral equation (11). Besides, (14) is simplified by the absence of $|k_{\rm B}^{-}\, k'^{+}|^{\alpha_{\rm R}}$ and $\mathcal{B}\{{\rm q}'\}$.

6 Application with string amplitudes

An *ab initio* string hadronization amplitude [5] can be expressed in the multiperipheral form with the propagator and vertex

$$\mathcal{D}\{{\rm q}\} = (k^- k^+ - i0)^{\alpha\{{\rm q}\}} \exp\left[(i-b)\, k^- k^+/2\right] d\{{\rm q}\}\,, \tag{18}$$

$$\mathcal{V}\{{\rm q}',{\rm h},{\rm q}\} = (p^+/k'^+)^{\alpha\{{\rm q}'\}} \exp\left[(b-i)\, k'^- k^+/2\right] (-p^-/k^-)^{\alpha\{{\rm q}\}}\, g\{{\rm q}',{\rm h},{\rm q}\}\,. \tag{19}$$

$d\{{\rm q}\} = d(f,\mathbf{k}_{\rm T})$ and $g\{{\rm q}',{\rm h},{\rm q}\} = g_{f',h,f}(\mathbf{k}'_{\rm T},\mathbf{k}_{\rm T})$ are spin matrices and $\alpha\{{\rm q}\} = \alpha(f,\mathbf{k}_{\rm T}^2)$. In the ladder approximation one can remove the phases of the exponential factors and of $(k^- k^+ - i0)^{\alpha\{{\rm q}\}}$. This does not change the probabilities. After renormalization,

$$\mathcal{V}\{{\rm q}',{\rm h},{\rm q}\} = (k'^+/p^+)^{a\{{\rm q}'\}/2} \exp(b\, k^+ k'^-/2)\, (-k^-/p^-)^{a\{{\rm q}\}/2}\, g\{{\rm q}',{\rm h},{\rm q}\}\,, \tag{20}$$

with a new $g\{{\rm q}',{\rm h},{\rm q}\}$ and $a\{{\rm q}\} =$ old $(\alpha_{\rm R} - 2\,{\rm Re}\,\alpha\{{\rm q}\})$. The right Eq.(17) becomes

$$U\{{\rm q}\} = \mathcal{E}(a\{q\}, -k^- k^+)\, u\{{\rm q}\} \quad \text{with} \quad \mathcal{E}(a,x) \equiv x^a\, e^{-bx}\,, \tag{21}$$

$$u\{{\rm q}\} = \sum_{h,\,s_{\rm h}} \int {\rm d}^2\mathbf{p}_{\rm T}\, \frac{dZ}{Z} \left(\frac{1-Z}{Z}\right)^{a\{{\rm q}'\}} \mathcal{E}\left(-a\{q\}, \frac{m_{\rm h}^2 + \mathbf{p}_{\rm T}^2}{Z}\right) g^\dagger\{{\rm q}',{\rm h},{\rm q}\}\, g\{{\rm q}',{\rm h},{\rm q}\}\,. \tag{22}$$

Example: $a\{{\rm q}\} = \textit{constant}$ and

$$g\{{\rm q}',{\rm h},{\rm q}\} = e^{-B\,(\mathbf{k}'^2_{\rm T} + \mathbf{k}_{\rm T}{}^2)}\, (\mu_{f'} + \sigma_z\, \sigma\cdot\mathbf{k}'_{\rm T})\; \Gamma\; (\mu_f + \sigma_z\, \sigma\cdot\mathbf{k}_{\rm T}) \tag{23}$$

with $\Gamma = \sigma_z$ for a pseudoscalar meson and $\Gamma = G_{\rm L}\, V_z^*\, \mathbf{I} + G_{\rm T}\, \sigma\cdot V_{\rm T}^*\, \sigma_z$ for a vector meson, like in the "toy model " [1]. A complex μ_f with ${\rm Im}\mu_f > 0$ reproduces the effects of the *string* $+^3P_0$ mechanism.

The receipe (14-15) becomes

1. generate the species and momentum of h following the distribution

$${\rm d}^2\mathbf{p}_{\rm T}\, \frac{dZ}{Z} \left(\frac{1-Z}{Z}\right)^{a\{{\rm q}'\}} \mathcal{E}\left(-a\{q\}, \frac{m_{\rm h}^2 + \mathbf{p}_{\rm T}^2}{Z}\right) \sum_{s_{\rm h}} {\rm Tr}\left(t\{{\rm q}',{\rm h},{\rm q}\}\; \rho\{{\rm q}\}\; t^\dagger\{{\rm q}',{\rm h},{\rm q}\}\right) \tag{24}$$

with $t\{{\rm q}',{\rm h},{\rm q}\} = g\{{\rm q}',{\rm h},{\rm q}\}\, u^{-\frac{1}{2}}\{{\rm q}\}$,

2. calculate the polarization of $\uparrow{\rm q}'$ with

$$\rho\{{\rm q}'\} = \big[\sum_{s_{\rm h}} t\{{\rm q}',{\rm h},{\rm q}\}\; \rho\{{\rm q}\}\, t^\dagger\{{\rm q}',{\rm h},{\rm q}\}\big] / \;{\rm Tr}\;\big[{\rm idem}\big]. \tag{25}$$

If quark spin is ignored, $g\{{\rm q}',{\rm h},{\rm q}\} = g_{f',h,f}(\mathbf{k}'^2_{\rm T}, \mathbf{p}^2_{\rm T}, \mathbf{k}^2_{\rm T})$, $u\{{\rm q}\} = u(f,\mathbf{k}^2_{\rm T})$ and one recovers the symmetric Lund model. $U\{{\rm q}\}$ and $\langle j|\, \mathcal{V}^\dagger\{{\rm q}',{\rm h},{\rm q}\}\, |j'\rangle\, \langle i'|\, \mathcal{V}\{{\rm q}',{\rm h},{\rm q}\}\, |i\rangle$ are the spin-dependent generalizations of $\rho_\nu(V)$ and $\rho_{\nu,\nu'}(V,V')$ in [10].

7 Conclusion

We have built a *bona fide* recursive quark fragmentation model including the quark spin degree of freedom. For pseudo-scalar and vector mesons the model can reproduce the Collins effects of the classical $string+{}^3P_0$ mechanism and also give longitudinal jet handedness. It can be a guide for quark polarimetry and may also account for "hidden spin" effects in unpolarized quark fragmentation. The *ab initio* input consists in quark propagators and vertices. Using it, an integral equation has to be solved in order to fix the splitting distribution. Starting from the *renormalized input*, which consists in exponents $a\{\mathrm{q}\} = a(f, \mathbf{k}_\mathrm{T}^2)$ and vertex matrices $g\{\mathrm{q}', \mathrm{h}, \mathrm{q}\} = g_{f',h,f}(\mathbf{k}'_\mathrm{T}, \mathbf{k}_\mathrm{T})$, only an ordinary integration is needed. Putting vertices derived from the semiclassical string action in 1+1 dimension, one obtains a spin-dependent generalization of the symmetric Lund model which may be implemented in a Monte-Carlo code of quark jet simulation.

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